7.7 Solve Right Triangles

Before You used tangent, sine, and cosine ratios.
Now You will use inverse tangent, sine, and cosine ratios.
Why? So you can build a saddle rack, as in Ex. 39.

To SOLVE A RIGHT TRIANGLE means to find the measures of all the sides **and** ∠s.

All the information that you need to solve a right triangle are:

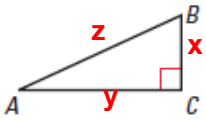
EX 1. 2 sides
EX 3. 2. 1 side, 1 acute ∠

USE: Looking for an ∠

KEY CONCEPT *For Your Notebook*

Inverse Trigonometric Ratios

Let $\angle A$ be an acute angle.



Inverse Tangent If $\tan A = \frac{x}{y}$, then $\tan^{-1}\frac{x}{y} = m\angle A$. $\tan^{-1}\frac{x}{y} = m\angle A$

Inverse Sine If $\sin A = \frac{x}{z}$, then $\sin^{-1}\frac{x}{z} = m\angle A$. $\sin^{-1}\frac{x}{z} = m\angle A$

Inverse Cosine If $\cos A = \frac{y}{z}$, then $\cos^{-1}\frac{y}{z} = m\angle A$. $\cos^{-1}\frac{y}{z} = m\angle A$

EXAMPLE 1 Use an inverse tangent to find an angle measure.

Use a calculator to approximate the measure of $\angle A$ to the nearest tenth of a degree.

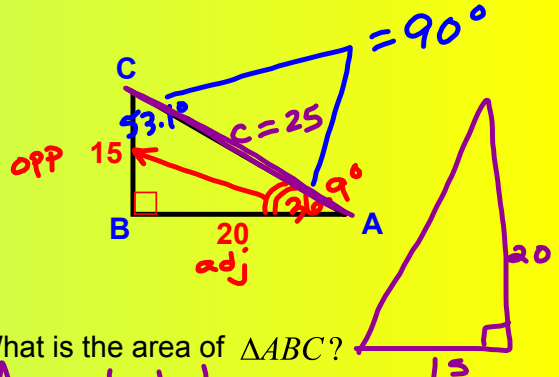
1. What is $m\angle A$? opp ∧ adj $\tan^{-1}\frac{15}{20} \approx 36.9°$

2. How can you find $m\angle C$? $90 - 36.9 = 53.1°$

3. What is the length of AC? $15^2 + 20^2 = c^2$, $225 + 400 = c^2$, $\sqrt{625} = c^2$, $25 = c$

4. What is the perimeter of ΔABC? $P = 15 + 20 + 25 = 60$ units

5. What is the area of ΔABC? $A_\Delta = \frac{1}{2} \cdot b \cdot h$, $A_\Delta = \frac{1}{2} \cdot 20 \cdot 15$, $A_\Delta = 150$ units^2



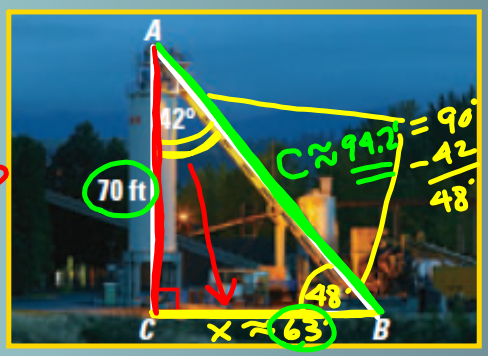
EXAMPLE 2 Use an inverse sine and an inverse cosine.

1. $\sin A = 0.87$ $m\angle A = \sin^{-1}.87 \approx 60.5°$

2. $\cos B = 0.15$ $m\angle B = \cos^{-1}.15 \approx 81.4°$

EXAMPLE 3 **Solve a right triangle.**

Solve the right triangle. Round decimal answers to the nearest tenth.



$\tan 42^\circ = \frac{x}{70}$ (opp, adj)

$70 \tan 42^\circ \approx x$

$63' \approx x$

adj, hyp

$70^2 + 63^2 = c^2$

$\sqrt{8869} = \sqrt{c^2}$

$94.2' \approx c$

$\cos 42^\circ = \frac{70}{c}$

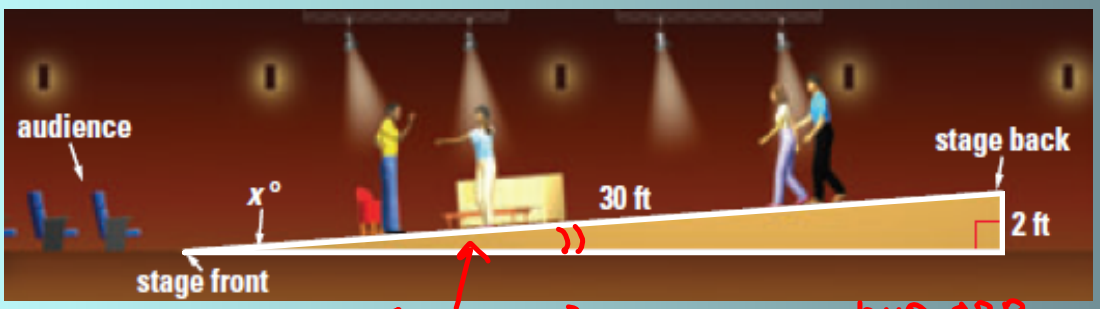
$c \cos 42^\circ = 70$

$\frac{c \cos 42^\circ}{\cos 42^\circ} = \frac{70}{\cos 42^\circ}$

$c \approx 94.2'$

EXAMPLE 4 **Solve a real-world problem.**

THEATER DESIGN Suppose your school is building a raked stage. The stage will be 30 feet long from front to back, with a total rise of 2 feet. A rake (angle of elevation) of 5° or less is generally preferred for the safety and comfort of the actors. Is the raked stage you are building within the range suggested?



$0^\circ \le x \le 5^\circ$

hyp, opp

SOH

$\sin^{-1} \frac{2}{30} \approx 3.8^\circ$

Safe!